Product Rule

$$\frac{d}{dx}[u\cdot v] = \frac{du}{dx}v + u\frac{dv}{dx}$$

Reverse Product Rule / Integration by Parts

$$\int \frac{d}{dx}[u\cdot v]\,dx = \int v\cdot\frac{du}{\cancel{dx}}\cancel{dx} + \int u\cdot\frac{dv}{\cancel{dx}}\cdot\cancel{dx}$$

$$u\cdot v - \int u'v = \int uv'\,dx \iff \boxed{\int u\,dv = uv - \int v\,du}$$

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ex Evaluate $\int x\cdot\cos(x)\,dx$

$u = x \quad v = \sin(x)$

$du = 1 \quad dv = \cos(x)$

Recall:

$$\int u\,dv = uv - \int v\,du$$

$$\int x\cos(x)\,dx = x\cdot\sin(x) - \int 1\cdot\sin(x)$$

$$= \boxed{x\cdot\sin(x) + \cos(x) + C}$$

What if we chose different initial u, dv??

★ Choosing u

- **L**ogarithm
- **I**nverse trig
- **P**olynomials
- **E**xponentials
- **T**rig

*Caution: Not for U-Substitution

The tricky thing is determining u, dv.
Integration by Parts does not always work.
Goal is to go from a hard Integral ($\int u\,dv$) to an easier one ($\int v\,du$).

Area Ex

$\int_0^3 xe^{-x}\,dx$

$u = x \quad v = -e^{-x}$

$du = 1\cdot dx \quad dv = e^{-x}$

$$\int u\,dv = uv - \int v\,du$$

$$= -xe^{-x} - \int -e^{-x}\cdot 1dx$$

$$= \left[-xe^{-x} - e^{-x} + C\right]_0^3 = (-3e^{-3} - e^{-3}) - (-1)$$

$$= \boxed{-3e^{-3} - e^{-3} + 1} = 0.80$$

lets approach $\int \ln(x)\,dx$ from different directions

Evaluate $\frac{d}{dx}[x\ln(x) - x]$

Recall: $u'v + uv'$

$1\cdot\ln(x) + x\cdot\frac{1}{x}$

$\ln(x) + 1$

$\Rightarrow \ln(x) + 1 - 1$

$\Rightarrow \boxed{\ln(x)}$

Evaluate $\int \ln(x)\,dx$

$u = \ln(x) \quad v = x$

$du = \frac{1}{x}\,dx \quad dv = dx$

Recall: $\int u\,dv = uv - \int v\,du$

$$\int \ln(x)\,dx = x\cdot\ln(x) - \int x\cdot\frac{1}{x}\,dx$$

$$\boxed{= x\cdot\ln(x) - x + C}$$

Using Int. by Parts more than once! "Repeated Use"

ex Evaluate $\int x^2 e^x\,dx$

$u = x^2 \qquad v = e^x$
$du = 2x\,dx \qquad dv = e^x$

Recall: $\int u\,dv = uv - \int v\,du$

$\int x^2 e^x = x^2 e^x - \int e^x \cdot 2x\,dx$

$= x^2 e^x - [\ ?\ ]$

Evaluate $\int e^x \cdot 2x\,dx$

$u = 2x \qquad v = e^x$
$du = 2\,dx \qquad dv = e^x\,dx$

Recall: $\int u\,dv = uv - \int v\,du$

$\int 2x\,e^x\,dx = 2xe^x - \int 2e^x\,dx$

$\boxed{= 2xe^x - 2e^x + C}$

$= x^2 e^x - [2xe^x - 2e^x + C]$

$\boxed{= x^2 e^x - 2xe^x + 2e^x + C}$

★ Final Answer

★ Be consistent with your u,v choice when doing Repeated use
Looking @ previous ex: We kept u with the x, and v with the e^x. DONT SWITCH!

ex Evaluate $\int e^x \cos(x)\,dx$

$u = e^x \qquad v = \sin(x)$
$du = e^x\,dx \qquad dv = \cos(x)\,dx$

Recall: $\int u\,dv = uv - \int v\,du$

$\int e^x \cos(x)\,dx = e^x \sin(x) - \int \sin(x)\,e^x\,dx$

$= e^x \sin(x) - [\ ?\ ]$

$\int \sin(x)\,e^x\,dx$

$u = e^x \qquad v = -\cos(x)$
$du = e^x\,dx \qquad dv = \sin(x)\,dx$

Recall: $\int u\,dv = uv - \int v\,du$

$\int e^x \sin(x)\,dx = \boxed{-e^x\cos(x) + \int e^x \cos(x)\,dx}$

$= e^x \sin(x) - [-e^x \cos(x) + \int e^x \cos(x)\,dx]$

$\int e^x \cos(x)\,dx = e^x \sin(x) + e^x \cos(x) - \int e^x \cos(x)\,dx$

$\Rightarrow 2\int e^x \cos(x)\,dx = e^x \sin(x) + e^x \cos(x)$

$\Rightarrow \int e^x \cos(x)\,dx = \boxed{\frac{e^x \sin(x) + e^x \cos(x)}{2} + C}$

★ Final Ans.

Tabular Integration

If we have $\int f(x)\, g(x)\, dx$ where $f(x)$ can be differentiated repeatedly to become zero, and $g(x)$ can be integrated repeatedly forever without going to zero.

ex) $\int x^2 e^x\, dx$

$f(x) = x^2 \qquad g(x) = e^x$

$f'(x)$	sign	$\int g(x)$
x^2	+	e^x
$2x$	−	e^x
2	+	e^x
0		e^x

$$\Rightarrow \boxed{\int x^2 e^x\, dx = x^2 e^x - 2x e^x + 2e^x + C}$$

ex) $\int x^3 \sin(x)\, dx$

$f(x) = x^3 \qquad g(x) = \sin(x)$

$f'(x)$	sign	$\int g(x)$
x^3	+	$\sin(x)$
$3x^2$	−	$-\cos(x)$
$6x$	+	$-\sin(x)$
6	−	$\cos(x)$
0		$\sin(x)$

$$\Rightarrow \boxed{\int x^3 \sin(x)\, dx = -x^3\cos(x) + 3x^2\sin(x) + 6x\cos(x) - 6\sin(x) + C}$$

Book HW

1-4, 9-14, 23

(9) $\int x^3 \ln(x)\,dx$

$u = \ln(x)$ $\quad v = \frac{1}{4}x^4$

$du = \frac{1}{x}$ $\quad dv = x^3$

$$\int x^3 \ln(x)\,dx = \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^4 \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{4}x^4 \ln(x) - \frac{1}{4}\int x^3\,dx \quad = \boxed{\frac{1}{4}x^4 \ln(x) - \frac{1}{16}x^4 + C}$$

(10) $\int x^4 e^{-x}\,dx$

$f'(x)$	sign	$\int g(x)$
x^4		e^{-x}
$4x^3$	$-$	$-e^{-x}$
$12x^2$	$+$	e^{-x}
$24x$	$-$	$-e^{-x}$
24	$+$	e^{-x}
		$-e^{-x}$

$$= \boxed{-x^4e^{-x} - 4x^3e^{-x} - 12x^2e^{-x} - 24e^{-x} - 24e^{-x} + C}$$

(11) $\int (x^2-5x)e^x\,dx$

$f'(x)$	Sign	$\int g(x)$
x^2-5x	$+$	e^x
$2x-5$	$-$	e^x
2	$+$	e^x
0	$-$	e^x

$$= \boxed{(x^2-5x)e^x - (2x-5)e^x + 2e^x + C}$$

$$e^x x^2 - 5xe^x - 2xe^x + 5e^x + 2e^x$$

$$= e^x x^2 - 7xe^x + 7e^x + C$$

☆ 13 $\int e^y \sin(y)\, dy$

$u = e^y \qquad v = -\cos(y)$

$du = e^y \qquad dv = \sin(y)$

$\Rightarrow -e^y\cos(y) + \int e^y \cos(y)\, dy$

$u = e^y \qquad v = \sin(y)$

$du = e^y \qquad dv = \cos(y)$

$\Rightarrow e^y\sin(y) - \int e^y \sin(y)\, dy$

$\Rightarrow \int e^y \sin(y)\, dy = -e^y\cos(y) + e^y\sin(y) - \int e^y\sin(y)\, dy$

$\Rightarrow 2\int e^y \sin(y)\, dy = -e^y\cos(y) + e^y\sin(y)$

Thus,

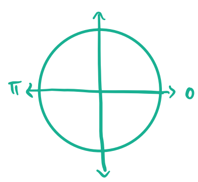
$$\boxed{\int e^y \sin(y)\, dy = \frac{-e^y\cos(y) + e^y\sin(y)}{2} + C}$$

23 a. $\int_0^{\pi} x\sin(x)\, dx$

$u = x \qquad v = -\cos(x)$

$du = 1 \qquad dv = \sin(x)$

$= -x\cos(x) + \int \cos(x)\, dx \quad = \Big[-x\cos(x) + \sin(x)\Big]_0^{\pi} = \boxed{\pi}$



ex $\int \arctan(x)\, dx$

$u = \arctan(x) \qquad v = x$

$du = \frac{1}{x^2+1}dx \qquad dv = 1$

$\Rightarrow x\cdot\arctan(x) - \int \frac{x}{x^2+1}\, dx$

$\Rightarrow \boxed{x\cdot\arctan(x) - \frac{1}{2}\cdot\ln|x^2+1| + C}$

1. Evaluate $\int 3x cos\,(3x)\,dx$.

   (a) $x cos\,(3x) - \frac{1}{3} sin\,(3x) + C$
   (b) $\frac{1}{3} x cos\,(3x) - 3x sin\,(3x) + C$
   (c) $\frac{1}{9} x sin\,(3x) + C$
   (d) $x sin\,(3x) + \frac{1}{3} cos\,(3x) + C$
   (e) $3 sin\,(3x) + C$

$u = 3x \qquad v = \frac{1}{3}\sin(3x)$

$du = 3 \qquad dv = \cos(3x)$

$x\sin(3x) - \int \sin(3x)\,dx$

$\Rightarrow x\sin(3x) + \frac{1}{3}\cos(3x) + C$

2. Evaluate $\int e^x sin\,(x)\,dx$

   (a) $-e^x cos\,(x) + C$
   (b) $e^x\,(sin\,(x) + cos\,(x)) + C$
   (c) $\frac{1}{2} e^x\,(cos\,(x) - sin\,(x)) + C$
   (d) $2e^x sin\,(x) + C$
   (e) $\frac{1}{2} e^x\,(sin\,(x) - cos\,(x)) + C$

(See notes)

3. Evaluate $\int \frac{x^{2}}{(x^3+1)^3}dx$.

   (a) $\frac{x^3}{12\,(x^3+1)^4} + C$
   (b) $\frac{x^3}{6\,(x^3+1)^2} + C$
   (c) $-\frac{1}{12\,(x^3+1)^4} + C$
   (d) $-\frac{1}{6\,(x^3+1)^2} + C$
   (e) $\frac{1}{9\,(x^3+3)^3} + C$

$\left.\begin{aligned} u &= x^3+1 \\ du &= 3x^2\,dx \end{aligned}\right\} \Rightarrow \frac{1}{3}\int \frac{1}{u^3}\,du$

4. Evaluate $\int 3x sec^2\,(x)\,dx$.

$u = 3x$ $\quad$ $v = \tan(x)$

$du = 3\,dx$ $\quad$ $dv = \sec^2(x)$

(a) $\frac{3}{2}x^2 tanx + C$

(b) $3xtanx + C$

(c) $3xln|secx| - 3tanx + C$

(d) $3xtanx - 3ln|secx| + C$

(e) $\frac{3}{2}x^2 tanx - 3ln|secx| + C$

$3x\tan(x) - 3\int \tan(x)\,dx$

$= 3x\tan(x) - 3\ln|\sec(x)| + c$

5. Evaluate $\int \frac{2x+3}{x}dx$.

(a) $-\frac{(x^2+3x)+C}{x^2}$

(b) $-(x^2+3x)\,ln|x| + C$

(c) $3ln|x| - 2x + C$

(d) $x^2 + 3x + 3ln|x| + C$

(e) $2x + 3ln|x| + C$

$= \int 2\,dx + 3\int \frac{1}{x}\,dx$

6. Evaluate $\int x^2 e^{-x} dx$.

(a) $e^{-x}\left(2 + 2x - x^2\right) + C$

(b) $\frac{1}{3}x^3 e^{-x} + C$

(c) $-e^{-x}\left(x^2 + 2x + 2\right) + C$

(d) $-\frac{1}{3}x^3 e^{-x} + C$

(e) $e^{-x}\left(x^2 - 2x - 2\right) + C$

(See notes)

$u = \sec(x)$ $\quad$ $v = \tan(x)$

$du = \sec(x)\cdot\tan(x)$ $\quad$ $dv = \sec^2(x)$

7. Evaluate $\int sec^3 x dx.$ $= \int \sec^2(x)\cdot\sec(x)\,dx$

$\Rightarrow \tan(x)\cdot\sec(x) - \int \sec(x)\cdot\tan^2(x)\,dx$

$\Rightarrow \tan(x)\cdot\sec(x) - \int \sec(x)\cdot(\sec^2(x)-1)\,dx$

$\Rightarrow \tan(x)\cdot\sec(x) + \underbrace{\int \sec(x)dx}_{\ln(\sec(x)+\tan(x))} - \int \sec^3(x)\,dx$

(a) $ln|secxtanx|^2 + C$

(b) $\frac{1}{4}ln|secx + tanx|^4 + C$

(c) $\frac{1}{2}(secxtanx + ln|secx + tanx|) + C$

(d) $\frac{1}{2}ln|secx + tanx| + C$

(e) $\frac{1}{2}(secxtanx) + C$

8. Evaluate $\int \left(x^4 + \frac{x}{9 + x^4}\right) dx.$

$\int \frac{x}{(x^2)^2+3^2}\,dx = \frac{1}{6}\arctan\left(\frac{x^2}{3}\right) + C$

(a) $\frac{1}{5}x^5 + tan^{-1}x + C$

(b) $\frac{1}{5}x^5 + \frac{1}{6}tan^{-1}\left(\frac{x^2}{3}\right) + C$

(c) $\frac{1}{5}x^5 + sin\left(\frac{x^2}{3}\right) + C$

(d) $\frac{1}{5}x^5 + \frac{1}{3}tan^{-1}\left(\frac{x^2}{3}\right) + C$

(e) $\frac{1}{5}x^5 + \frac{1}{9}tan^{-1}\left(\frac{x}{9}\right) + C$

9. Evaluate $\int e^x(1 - e^{2x})\,dx$ $= \int e^x - e^{3x}\,dx = e^x - \frac{1}{3}e^{3x} + C$

(a) $e^x - \frac{1}{3}e^{3x} + C$

(b) $(e^{x-1})(1 - e^{2x}) + C$

(c) $e^x - 1 + C$

(d) $\frac{1}{3}e^x(1 - e^{2x}) + C$

(e) $e^x - e^{3x} + C$

10. Evaluate $\int ln\left(e^{(x^2-x+1)}\right)dx$ $= \int x^2 - x + 1 \, dx$

(a) $ln|e^{x^2-x+1}| + C$

(b) $\frac{1}{3}x^3 - \frac{1}{2}x^2 + x + C$

(c) $x^2 - x + 1 + C$

(d) $e^{(x^2-x+1)}\left[ln|e^{x^2-x+1}| - 1\right] + C$

(e) $e^{(x^2-x+1)} + C$

11. Evaluate $\int sin^4 x dx$ $= \int (\sin^2(x))^2 dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right)^2 dx$

(a) $\frac{1}{4}\left(x - \frac{1}{2}sin(2x)\right) + C$

(b) $-cosxsin^3x - \frac{1}{2}sin(2x) + C$

(c) $-cosxsin^3x + \frac{3x}{2} - \frac{3sin(2x)}{4} + C$

(d) $-\frac{1}{4}cosxsin^3x + \frac{3x}{8} - \frac{3}{8}sinxcosx + C$

(e) $-\frac{1}{4}\left(cosxsin^3x + \frac{3x}{2} + 3sinxcosx\right) + C$

$= \int \frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{4}\cos^2(2x) \, dx$

$= \frac{x}{4} - \frac{1}{4}\sin(2x) + \frac{1}{4}\int \cos^2(2x) dx$

$= \frac{1}{4}\left(x - \sin(2x) + \frac{x}{2} + \frac{1}{8}\sin(4x)\right) + C$

$= \frac{3}{8}x - \frac{1}{2}\sin(x)\cdot\cos(x) + \frac{1}{8}\sin(x)\cdot\cos(x)\cdot\cos(2x)$

$= \frac{3}{8}x - \frac{1}{2}\sin(x)\cdot\cos(x) + \frac{1}{8}\sin(x)\cdot\cos(x)\cdot(1 - 2\sin^2(x))$

12. Evaluate $\int \frac{lnx}{4x}dx$.

(a) $\frac{1}{8}(lnx)^2 + C$

(b) $\frac{1}{4}ln\left(x^2\right) + C$

(c) $\frac{1}{4}(lnx)^2 + C$

(d) $\frac{1}{4}xln|x|\left(ln|x| - 1\right) + C$

(e) $\frac{1}{4}ln|x|\left(ln|x| - 1\right) + C$

$\left.\begin{array}{l} u = \ln(x) \\ du = \frac{1}{x}dx \end{array}\right\} \Rightarrow \frac{1}{4}\int u \, du$

13. Find $\frac{dy}{dx}$ if $y = \int_x^{x^2} \left(t^2 - t + 1\right) dt$

(a) $2x^5 - 2x^3 - \frac{1}{2}x^2 - x + 1$
(b) $2x^5 - 2x^3 - x^2 + 3x - 1$
(c) $2x^5 - 2x^3 + x^2 - 3x + 1$
(d) $\frac{1}{3}x^6 - \frac{1}{2}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 - x$
(e) $\frac{1}{3}x^6 - \frac{1}{2}x^4 - \frac{1}{3}x^3 - \frac{3}{2}x^2 - x$

$\left[\frac{1}{3}t^3 - \frac{1}{2}t^2 + t\right]_x^{x^2}$

14. Evaluate $\int_0^1 x\,(x+1)^{\frac{1}{3}}\,dx$.

$u = x \qquad v = \frac{3}{4}(x+1)^{\frac{4}{3}}$

$du = 1\,dx \qquad dv = (x+1)^{\frac{1}{3}}$

(a) -0.591
(b) -0.321
(c) 0.231
(d) 0.321
(e) 0.591

15. Evaluate $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$.

(a) $2e\,(e-1)$
(b) $2\left(e^2 - 1\right)$
(c) $2\left(e^2 + 1\right)$
(d) $2e\,(e+1)$
(e) $2e^2$

$\left.\begin{array}{l} u = \sqrt{x} \\ du = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}\,dx \end{array}\right\} 2\int e^u\,du \Rightarrow \left[2e^{\sqrt{x}}\right]_1^4$

$dx = 2\sqrt{x}\,du$

$= 2u\,du$

$2e^2 - 2e$

$2e\,(e-1)$

16. Find $\frac{dy}{dx}$ if $y = \int_0^{x^2} \frac{1}{2} cost dt$.

(a) $xcosx^2$

(b) $xsinx^2$

(c) $2xcosx^2$

(d) $2xsinx^2$

(e) $xcos(2x)$

17. $\int_4^9 \frac{x+1}{\sqrt{x}} dx$ $= \int \frac{x}{\sqrt{x}} dx + \int \frac{1}{\sqrt{x}} dx$ $2x^{\frac{1}{2}}$

$\left.\begin{aligned} u &= \sqrt{x} \\ du &= \frac{1}{2} \cdot \frac{1}{\sqrt{x}} dx \end{aligned}\right\}$ $2\int_2^3 u^2 du + 2\left[\sqrt{x}\right]_4^9$

(a) $9\frac{1}{3}$ (b) $13\frac{1}{3}$ (c) $14\frac{2}{3}$ (d) $15\frac{1}{6}$ (e) $33\frac{1}{3}$

18. If $f(x) = \int_{\frac{\pi}{2}}^{x} tan^{-1} t dt$, find $f'\left(\frac{\pi}{6}\right)$ to the nearest thousandth. $= \arctan\left(\frac{\pi}{6}\right)$

(a) -0.4823 (b) 0.4823 (c) 0.5236 (d) 0.5774 (e) 1.486

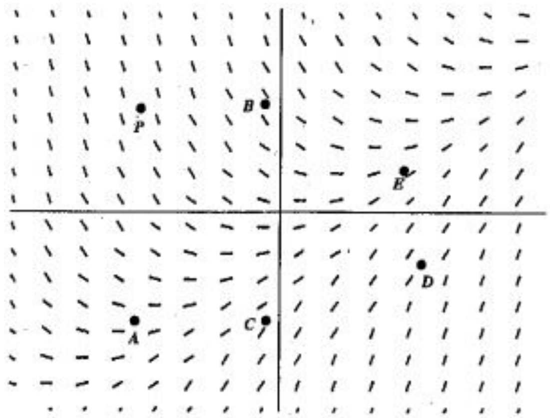
19. Evaluate $\int_{-1}^{1} (x^2 - 3)(x^5 + 2) dx$ $= \int_{-1}^{1} x^7 + 2x^2 - 3x^5 - 6 \; dx$

(a) $-13\frac{1}{3}$ (b) -12 (c) $-10\frac{2}{3}$ (d) $1\frac{1}{3}$ (e) $10\frac{2}{3}$

20. Evaluate $\int_0^{\frac{\pi}{6}} \sqrt{sinx} cosx dx$.

$\left.\begin{aligned} u &= \sin(x) \\ du &= \cos(x) \end{aligned}\right\} \Rightarrow \int \sqrt{u} \, du$

(a) $\frac{2\sqrt{3}}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{\sqrt{2}}{6}$ (d) $\frac{\sqrt{3}}{6}$ (e) $\frac{3\sqrt{2}}{8}$



1. A particular solution of the differential equation whose slope field is shown above contains point P. This solution may also contain which other point.

   (a) A (b) B (c) C (d) D (e) E

2. $\int_0^{\frac{\pi}{3}} sec^2 x tan^2 x dx$ equals

   $\left.\begin{array}{l} u = \tan(x) \\ du = \sec^2(x)\,dx \end{array}\right\} \Rightarrow \int u^2\,du$

   (a) $\frac{1}{3}$ (b) $\frac{\sqrt{3}}{3}$ (c) $\sqrt{3}$ (d) 3 (e) $3\sqrt{3}$

3. $\int_1^e ln\left(x\right) dx$ equals

   $\left[x\ln(x) - x\right]_1^e$

   (a) $\dfrac{1}{2}$ (b) $e-1$ (c) $e+1$ (d) 1 (e) -1

4. $\int_0^x f\left(t\right) dt = x sin\left(\pi x\right)$. Then $f\left(3\right) =$

   (a) -3π (b) -1 (c) 0 (d) 1 (e) 3π

5. $\displaystyle\int \frac{e^u}{1+e^{2u}}du$ is equal to

$u = e^u$
$du = e^u$ $\Big\} \Rightarrow \displaystyle\int \frac{1}{1+u^2}\,du$

(a) $ln\left(1+e^{2u}\right)+C$ (b) $\frac{1}{2}ln|1+e^u|+C$ (c) $\frac{1}{2}tan^{-1}e^u+C$

(d) $tan^{-1}e^u+C$ (e) $\frac{1}{2}tan^{-1}e^{2u}+C$

6. The minimum value of $f(x)=x^2+\frac{2}{x}$ on the interval $\frac{1}{2}\le x\le 2$ is

(a) $\dfrac{1}{2}$ (b) 1 (c) 3 (d) $4\dfrac{1}{2}$ (e) 5

$f'(x) = 2x - \frac{2}{x^2}$

$f'(1) = 0$

$f(1) = 3$

7. If $\displaystyle\int xcos(x)\,dx =$ (See notes)

(a) $xsin(x)+cos(x)+C$ (b) $xsin(x)-cos(x)+C$ (c) $\dfrac{x^2}{2}sin(x)+C$

(d) $\dfrac{1}{2}sin\left(x^2\right)+C$ (e) none of these

8. The only function that does not satisfy the Mean Value Theorem on the interval specified is

(a) $f(x)=x^2-2x$ on $[-3,1]$
(b) $f(x)=\frac{1}{x}$ on $[1,3]$
(c) $f(x)=\frac{x^3}{3}-\frac{x^2}{2}+x$ on $[-1,2]$
(d) $f(x)=x+\frac{1}{x}$ on $[-1,1]$
(e) $f(x)=x^{2/3}$ on $\left[\frac{1}{2},\frac{3}{2}\right]$

$\exists c \in [a,b]$ s.t.

$f'(c) = \dfrac{f(b)-f(a)}{b-a}$

9. If $\displaystyle\int_0^1 x^2e^x\,dx =$ (see notes)

(a) $-3e-1$ (b) $-e$ (c) $e-2$ (d) $3e$ (e) $4e-1$

$\int \frac{1}{H-70}\, dH = \int -0.05\, dt \Rightarrow \ln(H-70) = -0.05t + C \Rightarrow H-70 = H_0 e^{-0.05t}$

$\Rightarrow H(t) = 70 + 120 e^{-0.05t}$

$H(10) =$

10. A cup of coffee placed on a table cools at a rate of $\frac{dH}{dt} = -0.05\,(H-70)^{\circ}\, F$ per minute, where H represents the temperature of the coffee and t is time in minutes. If the coffee was at 120°F initially, what will its temperature be 10 minutes later? $H(10) =$

H_0

(a) $73^{\circ} F$ (b) $95^{\circ} F$ (c) $100^{\circ} F$ (d) $118^{\circ} F$ (e) $143^{\circ} F$

11. If $\sqrt{x-2}$ is replaced by u, then $\int_3^6 \frac{\sqrt{x-2}}{2} dx$ is equivalent to $\int_1^2$ du

(a) $\int_1^2 \frac{u du}{u^2+2}$ (b) $2\int_1^2 \frac{u^2 du}{u^2+2}$ (c) $\cancel{\int_3^6 \frac{2u^2}{u^2+2} du}$

(d) $\cancel{\int_3^6 \frac{u du}{u^2+2}}$ (e) $\frac{1}{2}\int_1^2 \frac{u^2}{u^2+2} du$

12. The table shows the depths of water, W, in a river, as measured at 4-hour intervals during a day-long flood. Assume that W is differentiable function of time t.

$t\,(hr)$	0	4	8	12	16	20	24
$W\,(t)\,(ft)$	32	36	38	37	35	33	32

$\frac{33-37}{20-12}$

(a) Find the approximate value of $W'(16)$. Indicate units of measure.

(b) Estimate the average depth of the water, in feet, over the time interval $0 \le x \le 24$ hours by using "The" trapezoidal approximation with subintervals of length $\Delta t = 4$ days. $4 \cdot \frac{1}{2}\,(32 + 2\cdot 36 + \cdots + 2\cdot 33 + 32)$

(c) Scientists studying the flooding believe they can model the depth of water with the function $F(t) = 35 - 3cos\left(\frac{t+3}{4}\right)$, where $F(t)$ represents the depth of the water, in feet, after t hours. Find $F'(16)$ and explain the meaning of your answer, with appropriate units, in terms of the river depth. $F'(t) = \frac{3}{4}\sin\left(\frac{t+3}{4}\right) \longrightarrow -0.75\ \frac{ft}{hr}$ water level is dropping

(d) Use the function F to find the average depth of the water, in feet, over the time interval $0 \le t \le 24$ hours. $\frac{1}{24-0} \cdot \int_0^{24} 35 - 3\cos\left(\frac{t+3}{4}\right) dt$

13. The amount of radiation $R(t)$ in a certain liquid decreases at a rate proportional to the amount present, that is $\frac{dR}{dt} = kR$, where k is a constant and t is measured in seconds. The initial amount of radiation is 10^6 rads. After 100 seconds the radiation has dropped to 10^2 rads.

R_0

(a) Express R as a function of t.

(b) To the nearest second, when will the amount of radiation has dropped below 10 rads?

(c) What is the half-life of this chemical? That is, how long does it take for the amount of radiation to reach half of the original amount?

(a) $\int \frac{1}{R}\, dR = \int k\, dt \Rightarrow R(t) = R_0 e^{kt} \Rightarrow 10^2 = 10^6 e^{100k}$

$k = \frac{\ln(10^{-4})}{100}$

$\Rightarrow R(t) = 10^6 e^{\left(\frac{\ln(10^{-4})}{100}\right) \cdot t}$

(b) $10 = 10^6 e^{\left(\frac{\ln(10^{-4})}{100}\right) \cdot t}$ Solve for t

(c) $10^3 = 10^6 e^{\left(\frac{\ln(10^{-4})}{100}\right) \cdot t}$ or $\frac{\ln(2)}{k} = \frac{100 \cdot \ln(2)}{\ln(10^{-4})}$